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Effects of hypersonic field and anharmonic interactions on channelling radiation

Juby George[1], **Anand P Pathak**[1,3], **L N S Prakash Goteti**[2] **and G Nagamani**[1]

[1] School of Physics, University of Hyderabad, Hyderabad-500 046, India
[2] Department of Computer Science and Information Systems, BITS-Pilani-333 031, India

E-mail: appsp@uohyd.ernet.in

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Abstract
The effects of a hypersonic field on positron channelling radiation are considered. Anharmonic effects of the transverse potential induced by these longitudinal fields are incorporated and the wavefunction of the planar channelled positron is found by the solution of Dirac equation under the resonant influence of hypersound. An expression for the resonant frequency is estimated. The transition probabilities and the intensity of the channelling radiation are also calculated. It is found that the anharmonic effects change the spectral distributions considerably.

1. Introduction

Investigations of radiation emitted by relativistic e^+ and e^-, channelled along major crystallographic directions, and the interaction of the channelled particle with this radiation field itself are of great value in atomic physics and accelerator based research. The underlying basic principle, namely that the accelerated charge should emit electromagnetic radiation, has been discussed from the very beginning, on the basis of classical electrodynamics. Since the oscillatory frequencies ω_0 of the channelled particles are low, the corresponding energies $\hbar\omega_0$ being of the order of a few eV only, the observation of this radiation appeared to be precluded. However, the realization that relativistic effects will shift the photon energy into the keV or even MeV region for MeV and GeV positrons and electrons, respectively, was a turning point. The radiation was in fact observed for the case of positrons about 20 years ago [1]. Thereafter, channelling radiation, being an interesting radiative phenomenon as a subject, has been investigated and reviewed by several authors [2]. Theory was formulated [3] in the relativistic quantum mechanical framework, and the experimental confirmation was made

[3] Author to whom any correspondence should be addressed.

initially for positrons [1] and later for electrons [4]. Apart from direct application, channelling radiation has opened possibilities of new applications in the fields of laser physics and medicine as a source of hard x-rays and γ-rays for nuclear pumping, and hence for possible construction of a γ-laser. The x-rays created as part of channelling even lead to new phenomena like photon channelling and x-ray optics [5].

The interactions of the electromagnetic fields with the lattice medium are another interesting area of current research. A few examples of such interactive phenomena are the well-known Cherenkov radiation and the transition radiation. The usefulness of these processes is to extract the maximum intensity of the emitted radiation. This requires optimization and proper understanding of the related conditions. An enhancement of radiation intensity can be achieved by the use of resonance effects between an external field (quite often an electromagnetic field) and the radiation process. The Dirac equation with a quasi-static transverse potential, followed by an analysis parallel to that of Kumakhov [3] using spinor components, resulted in the broadening of the radiation band width. Hypersonic/ultrasonic excitations of the medium also have the ability to produce such resonance effects. Most interestingly, this situation of ultra-relativistic positron radiation in a hypersonic wavefield excited in the crystallographic medium has been addressed very recently [6]. The hypersonic field results in compressions and rarefactions along the direction of propagation. In addition, the crystal potential in the transverse space makes the particle feel a periodic potential along the longitudinal direction. Grigorian *et al* [6, 7] treated this process in a harmonic approximation of the transverse planar potential. However, a more accurate description is required to include the actual influence of these acoustic fields on the already existing parabolic transverse potential. As a result, the shape of this transverse potential becomes more complex, and hence the resulting anharmonicity effects play a crucial role in the resonance process.

In the present work we investigate the influence of such anharmonicity effects due to acoustic longitudinal hypersonic oscillations excited in the lattice on the channelling radiation. In the next section the mathematical formulation of the process is given. The wavefunction for the planar channelling of a positron and the frequency of radiation in a hypersonic field is obtained in section 3. Section 4 is dedicated to the radiative transitions induced by hypersound. The transition probabilities and the intensity of radiation are obtained in section 5. Section 6 deals with inverse radiative transitions, and finally, in section 7, some concluding remarks are given.

2. Anharmonic effects on the wavefunction

The planar potential based on the Lindhard standard potential with slight modification [8] has been shown to be reasonable for both dechannelling calculations [9] and channelling radiation characteristics [10]. We continue to use this potential so that the planar potential due to both the planes surrounding a channel can be written as

$$V(x) = V_0 x^2 + V_1 x^4 \tag{1}$$

where

$$\begin{aligned} V_0 &= \frac{4\pi Z_1 Z_2 e^2 C a^2 N_{\mathrm{p}}}{(l+a)^3} \\ V_1 &= \frac{4\pi Z_1 Z_2 e^2 C a^2 N_{\mathrm{p}}}{(l+a)^5} \end{aligned} \tag{2}$$

where C is the Lindhard constant ($=\sqrt{3}$), a is the Thomas Fermi screening distance, Z_1 and Z_2 are the atomic numbers of the incident ion and target atoms respectively, $N_{\mathrm{p}} = N d_{\mathrm{p}}$ is the planar density of atoms, N being the bulk density of atoms in the crystal and d_{p} the interplanar spacing, and $l = \frac{d_{\mathrm{p}}}{2}$. The coefficients V_0 and V_1 are obtained by incorporating the appropriate crystal parameters and x is the position coordinate in the transverse space, measured from the mid-plane.

The initial investigations on channelling radiation [3] did not consider the anharmonic part of the potential shown in equation (1). Later, the effects of anharmonicity were considered [11]. The total energy spectrum is the sum of harmonic and anharmonic contributions, which can be written to the first order as

$$\varepsilon'_{nE} = \varepsilon_{nE} + \tfrac{3}{4} V_1 \alpha^4 (2n^2 + 2n + 1) \tag{3}$$

where

$$\varepsilon_{nE} = \hbar\omega_E \left(n + \frac{1}{2}\right), \qquad \omega_E = c\sqrt{\frac{2V_0}{E}} \qquad \text{and} \qquad \alpha = c\sqrt{\frac{\hbar}{E\omega_E}}. \tag{4}$$

The effects of the external hypersonic field on the planar potential 'seen' by the relativistic positron are incorporated as [12, 13]

$$U(x, z) = U_0 \cos(k_s z) + V(x)[1 + \mu \cos(k_s z)] \tag{5}$$

where $k_s = 2\pi/\lambda_s$, λ_s is the wavelength of the hypersonic wave, and U_0 and μ are the modulation parameters of the potential. The z-dependence of the potential arises since the hypersonic field is propagating in the z-direction, and reflects the periodic compression and rarefaction in the medium. Hence the equation of motion of the relativistic positron in this external field is given by the Dirac equation:

$$[(E - U)^2 - m_0^2 c^4 + \hbar^2 c^2 \nabla^2]\psi_E(r) = 0. \tag{6}$$

The above equation assumes that the spin effects are negligible; at these high energies ($>$10 MeV).

Without the acoustic field, the above Dirac equation is satisfied by the wavefunction,

$$\psi^{(0)}_{np_yE}(r) = \frac{1}{\sqrt{l_y l_z}} S_{nE}(x) \exp\left[\frac{\mathrm{i}}{\hbar}(p_y y + p_z z)\right], \tag{7}$$

where p_y and p_z are the projections of the momentum of the positron on the y-axis and z-axis respectively, and l_y and l_z denote the thickness of the crystal along the y and z coordinates. S_{nE} are the oscillator wavefunctions, defined by

$$S_{nE} = \frac{\exp\left(-\frac{x^2}{2\alpha^2}\right)}{\sqrt{2^n n! \alpha\sqrt{\pi}}} H_n\left(\frac{x}{\alpha}\right) \tag{8}$$

which satisfy the Schrödinger equation,

$$\left(-\frac{\hbar^2}{2M}\frac{\mathrm{d}^2}{\mathrm{d}x^2} + V(x)\right) S_{nE} = \varepsilon'_{nE} S_{nE} \qquad \text{where } M = \frac{E}{c^2}. \tag{9}$$

To find the positron wavefunction in the presence of a hypersonic field, we make the series expansion,

$$\psi_{np_yE}(r) = \exp\left(\frac{\mathrm{i}}{\hbar} p_y y\right) \sum_{k=0}^{\infty} C_{knE}(z) S_{kE}(x). \tag{10}$$

Substituting equation (10) in (6) and using the condition $|U| \ll E$, we get

$$-\left(\frac{E^2 - m_0^2c^4 - p_y^2c^2}{2E}\right)\psi_{np_yE}(r) + U\psi_{np_yE}(r) - \frac{\hbar^2c^2}{2E}\frac{\partial^2}{\partial x^2}\exp\left(\frac{\mathrm{i}}{\hbar}p_y y\right) \times \sum_{k=0}^{\infty} C_{knE}(z)S_{kE}(x) - \frac{\hbar^2c^2}{2E}\frac{\partial^2}{\partial z^2}\exp\left(\frac{\mathrm{i}}{\hbar}p_y y\right)\sum_{k=0}^{\infty} C_{knE}(z)S_{kE}(x) = 0. \quad (11)$$

To further simplify, we use the creation and annihilation operator representation for x as $(a + a^\dagger)\frac{\alpha}{\sqrt{2}}$. Here a (annihilation operator) and $a^\dagger$ (creation operator) satisfy the well-known general relations,

$$a\psi_n = \sqrt{n}\psi_{n-1}$$
$$a^\dagger\psi_n = \sqrt{n+1}\psi_{n+1}.$$

After some detailed algebra and simplifications, this leads to a system of recursive relation for the coefficients C_{knE} appearing in equation (10) as follows:

$$\begin{aligned} -\frac{\hbar^2c^2}{2E}\frac{\partial^2}{\partial z^2}C_k &+ \left[U_0\cos k_s z + \varepsilon'_{kE}\left(1 + \frac{\mu}{2}\cos k_s z\right)\right. \\ &+ \left.\frac{\mu}{2}\cos k_s z\frac{3}{4}V_1\alpha^4(2k^2 + 2k + 1) - \frac{E^2 - m_0^2c^4 - p_y^2c^2}{2E}\right]C_k \\ &+ \tfrac{1}{4}\mu\hbar\omega_E\cos k_s z[\sqrt{k(k-1)}C_{k-2} + \sqrt{(k+1)(k+2)}C_{k+2}] \\ &+ \tfrac{1}{4}\mu V_1\alpha^4\cos k_s z[\sqrt{k(k-1)(k-2)(k-3)}C_{k-4} \\ &+ \sqrt{(k+1)(k+2)(k+3)(k+4)}C_{k+4} + (4k-2)\sqrt{k(k-1)}C_{k-2} \\ &+ (4k+6)\sqrt{(k+1)(k+2)}C_{k+2}] = 0. \end{aligned} \quad (12)$$

The evaluation of the unknown coefficients, C_k, is carried out by expanding them in terms of plane waves along the direction of motion of the positron (i.e. the z-axis) [6]:

$$C_k(z) = A_k(z)\exp(\mathrm{i}B_k(z)) \quad (13)$$

$$\hbar B_k(z) = p_{kE}z + \int \sigma_k(z)\,\mathrm{d}z. \quad (14)$$

The values of σ_k are the solutions of the equation

$$\mathrm{i}\hbar\frac{\mathrm{d}\sigma_k}{\mathrm{d}z} = \sigma_k(\sigma_k + 2p_{kE}) + \frac{E}{c^2}(2U_0 + \mu\varepsilon'_{kE})\cos k_s z \quad (15)$$

and

$$p_{kE}c = \sqrt{E^2 - m_0^2c^4 - p_y^2c^2 - 2E\varepsilon'_{kE}}. \quad (16)$$

Since $|U| \ll E$ and $k_s \leqslant \omega_E$, we can use the approximation $|\frac{\mathrm{d}F_k}{\mathrm{d}z}| \approx k_s|F_k| \leqslant \omega_E F_k$ where $F_k = \sigma_k, A_k$, and using $\frac{\sigma_k}{p_{kE}} \ll 1$ we have

$$\sigma_k = -\varsigma_{\mathrm{ah}}(U_0 + \tfrac{1}{2}\mu\varepsilon'_{kE})\cos k_s z \quad (17)$$

where

$$\varsigma_{\mathrm{ah}} = \frac{E}{p_{kE}c^2}.$$

Substituting the above equations in equation (12) and rearranging, we get

$$
\begin{aligned}
\mathrm{i}\frac{\mathrm{d}A_k}{\mathrm{d}z} = \varsigma_{\mathrm{ah}}\Bigg\{\exp(-\mathrm{i}B_k)\Bigg[&\frac{1}{4}\mu\omega_E\cos k_s z[\sqrt{k(k-1)}A_{k-2}\exp(\mathrm{i}B_{k-2})\\
&+\sqrt{(k+1)(k+2)}A_{k+2}\exp(\mathrm{i}B_{k+2})]\\
&+\tfrac{1}{4}\mu V_1\alpha^4\cos k_s z[\sqrt{k(k-1)(k-2)(k-3)}A_{k-4}\exp(\mathrm{i}B_{k-4})\\
&+\sqrt{(k+1)(k+2)(k+3)(k+4)}A_{k+4}\exp(\mathrm{i}B_{k+4})\\
&+(4k-2)\sqrt{k(k-1)}A_{k-2}\exp(\mathrm{i}B_{k-2})\\
&+(4k+6)\sqrt{(k+1)(k+2)}A_{k+2}\exp(\mathrm{i}B_{k+2})]\Bigg]\\
&+\frac{\mu}{2}\cos k_s z\frac{3}{4}\frac{V_1\alpha^4}{\hbar}(2k^2+2k+1)A_k\Bigg\}.
\end{aligned} \tag{18}
$$

Also we can rewrite equation (14) as

$$\hbar B_k = p_{kE}z - \varsigma_{\mathrm{ah}}\left(U_0 + \frac{1}{2}\mu\varepsilon'_{kE}\right)\frac{\sin k_s z}{k_s}. \tag{19}$$

Introducing the column matrix,

$$\hat{A} = \begin{pmatrix} A_1 \\ A_2 \\ \cdots \\ \cdots \end{pmatrix}, \tag{20}$$

equation (18) can be written in compact form

$$\mathrm{i}\frac{\mathrm{d}\hat{A}}{\mathrm{d}z} = \mu\hat{H}\hat{A} \tag{21}$$

with

$$\hat{H} = \begin{bmatrix} J & 0 & (u^*+6w^*)\sqrt{2} & 0 & v^*\sqrt{24} & \cdots \\ 0 & 5J & 0 & (u^*+10w^*)\sqrt{6} & 0 & \cdots \\ (u+6w)\sqrt{2} & 0 & 13J & 0 & (u^*+14w^*)\sqrt{12} & \cdots \\ 0 & (u+10w)\sqrt{6} & 0 & 25J & 0 & \cdots \\ v\sqrt{24} & 0 & (u+14w)\sqrt{12} & 0 & 41J & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \end{bmatrix}. \tag{22}$$

The above Hermitian matrix can be written in the form of recurrence relations given by

$$
\begin{aligned}
H_{k+2,k} &= H^*_{k,k+2} = [u + (4k+6)w]\sqrt{(k+1)(k+2)}\\
H_{k+4,k} &= H^*_{k,k+4} = v\sqrt{(k+1)(k+2)(k+3)(k+4)}\\
H_{k,k} &= J(2k^2+2k+1)
\end{aligned} \tag{23}
$$

where

$$J = \frac{3}{8}t\zeta_{\mathrm{ah}}\mu\cos k_s z, \qquad t = \frac{V_1\alpha^4}{\hbar}$$

and

$$
\begin{aligned}
u &= (1/4)\varsigma_{\mathrm{ah}}\omega_E \cos k_s z \exp\left(\mathrm{i}\varsigma_{\mathrm{ah}}\left(2\omega_E + \frac{3}{4}t(8k+12)\right)\left[z + \frac{\mu}{2}\frac{\sin k_s z}{k_s}\right]\right)\\
v &= (1/4)t\varsigma_{\mathrm{ah}} \cos k_s z \exp\left(2\mathrm{i}\varsigma_{\mathrm{ah}}\left(2\omega_E + \frac{3}{4}t(8k+20)\right)\left[z + \frac{\mu}{2}\frac{\sin k_s z}{k_s}\right]\right)\\
w &= (1/4)t\varsigma_{\mathrm{ah}} \cos k_s z \exp\left(\mathrm{i}\varsigma_{\mathrm{ah}}\left(2\omega_E + \frac{3}{4}t(8k+12)\right)\left[z + \frac{\mu}{2}\frac{\sin k_s z}{k_s}\right]\right).
\end{aligned}
\tag{24}
$$

The solution of equation (21) can be written as

$$\hat{A}(z) = \hat{Q}(z)\cdot\hat{R} \tag{25}$$

where $\hat{Q}(z) = \exp[-\mathrm{i}\mu\int\hat{H}(z)\,\mathrm{d}z] = 1 - \mathrm{i}\mu\int\hat{H}(z)\,\mathrm{d}z$ to the first order in μ, since μ is small.

The column matrix $\hat{R}$ is composed of integration constants R_k, normalized by the condition

$$\sum_k |R_k| = \frac{1}{\sqrt{l_y l_z}}.$$

The solution corresponding to different transverse states can be found by setting

$$R_k = \frac{\delta_{kn}}{\sqrt{l_y l_z}} \qquad n, k = 0, 1, 2\ldots.$$

The wavefunction of the positron is then written as

$$
\begin{aligned}
\psi_{np_yE} &= \frac{1}{\sqrt{l_y l_z}}\exp\left(\frac{\mathrm{i}}{\hbar}p_y y\right)\sum_k S_{kE}(x)Q_{kn}(z)\\
&\quad\times\ \exp\left(\frac{\mathrm{i}}{\hbar}\left[p_{kE}z - \varsigma_{\mathrm{ah}}\left(U_0 + \frac{1}{2}\mu\varepsilon'_{kE}\right)\frac{\sin k_s z}{k_s}\right]\right).
\end{aligned}
\tag{26}
$$

3. Resonant influence of hypersound

Due to the external hypersonic wave momentum $\hbar k_s$, the positron momentum p_z takes three possible values as

$$p_z = \begin{cases} p_{nE} - \hbar k_s \\ p_{nE} \\ p_{nE} + \hbar k_s. \end{cases} \tag{27}$$

Correspondingly, equation (26) can be used to write the superposed wavefunction as

$$\psi_{np_yE} = \psi^{(-1)}_{np_yE} + \psi^{(0)}_{np_yE} + \psi^{(+1)}_{np_yE} \tag{28}$$

where

$$\psi^{(0)}_{np_yE} = \frac{1}{\sqrt{l_y l_z}}S_{nE}(x)\exp\left\{\frac{\mathrm{i}}{\hbar}[p_y y + p_{nE}z]\right\}\left[1 - \mathrm{i}\frac{3}{8}\mu^2\varsigma_{\mathrm{ah}}t(2n^2+2n+1)\frac{\sin(k_s z)}{k_s}\right] \tag{29}$$

$$
\begin{aligned}
\psi^{(\varepsilon)}_{np_yE} &= \frac{1}{\sqrt{l_y l_z}}[b^{(\varepsilon)}_{n-4}S_{n-4,E}(x) + b^{(\varepsilon)}_{n-2}S_{n-2,E}(x)\\
&\quad + b^{(\varepsilon)}_{n}S_{n,E}(x) + b^{(\varepsilon)}_{n+2}S_{n+2,E}(x) + b^{(\varepsilon)}_{n+4}S_{n+4,E}(x)]\\
&\quad\times\ \exp\left\{\frac{\mathrm{i}}{\hbar}[p_y y + (p_{nE} + \varepsilon\hbar k_s)z]\right\}
\end{aligned}
\tag{30}
$$

with $\varepsilon = \pm 1$. During the above simplification the amplitudes are found to be

$$\begin{aligned}
b_{n-4}^{(\pm 1)} &= \frac{\mu t \zeta_{\rm ah}\sqrt{n(n-1)(n-2)(n-3)}}{8k_s(2\xi \mp 1)} \\
b_{n-2}^{(\pm 1)} &= \frac{\mu \zeta_{\rm ah}(\omega_E + (4n-2)t)\sqrt{n(n-1)}}{8k_s(\xi \mp 1)} \\
b_{n}^{(\pm 1)} &= \mp \frac{\zeta_{\rm ah}}{2\hbar k_s}\left(U_0 + \frac{1}{2}\mu \varepsilon'_{nE}\right) \\
b_{n+2}^{(\pm 1)} &= -\frac{\mu \zeta_{\rm ah}(\omega_E + (4n+6)t)\sqrt{(n+1)(n+2)}}{8k_s(\xi \pm 1)} \\
b_{n+4}^{(\pm 1)} &= -\frac{\mu t \zeta_{\rm ah}\sqrt{(n+1)(n+2)(n+3)(n+4)}}{8k_s(2\xi \pm 1)}
\end{aligned} \tag{31}$$

where $\xi = \frac{2\zeta_{\rm ah}\omega_E}{k_s}$.

In addition to the resonance obtained earlier for the harmonic case [6], it further follows from the above equation (31) that $b_{n-4}^{(+1)} \to b_{n+4}^{(-1)} \to \infty$ when $2\xi = \frac{4\zeta_{\rm ah}\omega_E}{k_s} \to 1$. Thus the absolute values of the amplitudes of the states with $p_z = p_{nE} \pm \hbar k_s$ show a sudden (resonance) increase when the wavelength $\lambda_s = \frac{2\pi}{k_s}$ decreases and reaches the critical value

$$\lambda_{\rm ah}^{\rm c} = \frac{\pi}{2\varsigma_{\rm ah}\omega_E} \tag{32}$$

which is just half of the corresponding resonance wavelength for the harmonic case. This leads to a variation in the intensity of the radiation, as discussed later in sections 5 and 6.

The radiation frequency obtained by the Doppler formula for the harmonic case is given by [6]

$$\omega_{\rm h} = \frac{\Omega_{fi} - vk_s\varsigma_{\rm h}^{-1}}{1 - \beta_{\parallel}\cos\theta} \tag{33}$$

where

$$\Omega_{fi} = \frac{E_{ni} - E_{nf}}{\hbar}.$$

A similar analysis to incorporate energy changes due to anharmonic effects leads to the modified formula for frequency as

$$\omega_{\rm ah} \approx \frac{\Omega_{fi}\left[1 + \left(\frac{3V_1}{4V_0}\right)\alpha^2(n_i + n_f + 1)\right] - vk_s\zeta_{\rm ah}^{-1}}{1 - \beta_{\parallel}\cos\theta}. \tag{34}$$

Thus the fractional change in the radiative frequency due to anharmonicity is given by

$$\frac{\Delta\omega}{\omega_{\rm h}} = \frac{\Omega_{fi}\left(\frac{3V_1}{4V_0}\right)\alpha^2(n_i + n_f + 1) - vk_s(\zeta_{\rm ah}^{-1} - \zeta_{\rm h}^{-1})}{\Omega_{fi} - \zeta_{\rm h}^{-1}}. \tag{35}$$

One can notice from the above expression that the fractional change in the resonance frequency $\to 0$ in the absence of anharmonic interactions.

4. Radiative transitions induced by hypersound

Let us consider the radiative transitions of the channelled positrons that are stimulated by the hypersound.

The probability of transitions from an initial state i to a final state f is given by the formula [14]

$$W_{fi} = \frac{4\pi^2 e^2}{\hbar V} \sum_{\vec{q}} |\vec{q}|^{-1} |\vec{\alpha}_{fi} \cdot \vec{e}_k|^2 \delta(\omega_{fi} - \omega) \tag{36}$$

where V is the volume of the system and $\vec{q}$ and $\vec{e}_k$ are the wavevector and polarization vector of the photon and the matrix elements $\vec{\alpha}_{fi}$ are given by

$$\vec{\alpha}_{fi} = \frac{1}{E} \delta_{\sigma_{iz},\sigma_{fz}} \int \mathrm{e}^{-\mathrm{i}\vec{q}\vec{r}} \psi_f^*(\vec{r}) \hat{\vec{p}} \psi_i(\vec{r})\, \mathrm{d}\vec{r}. \tag{37}$$

After integration over y and z we have the following expression for the matrix elements:

$$\vec{\alpha}_{fi} = \delta_{\sigma_{iz},\sigma_{fz}} \delta_{p_{iy},p_{fy}+\hbar q_y} [\vec{D}_{fi}^{(-2)} \delta_{p_{n_i E_i - 2\hbar k_s}, p_{n_f E_f + \hbar q_z}} + \vec{D}_{fi}^{(-1)} \delta_{p_{n_i E_i - \hbar k_s}, p_{n_f E_f + \hbar q_z}} \\ + \vec{D}_{fi}^{(0)} \delta_{p_{n_i E_i}, p_{n_f E_f + \hbar q_z}} + \vec{D}_{fi}^{(+1)} \delta_{p_{n_i E_i + \hbar k_s}, p_{n_f E_f + \hbar q_z}} + \vec{D}_{fi}^{(+2)} \delta_{p_{n_i E_i + 2\hbar k_s}, p_{n_f E_f + \hbar q_z}}] \tag{38}$$

where

$$\vec{D}_{fi}^{(\nu)} = \frac{1}{E} \sum_{\varepsilon_1,\varepsilon_2} \delta_{\nu,\varepsilon_2-\varepsilon_1} \int \exp(-\mathrm{i} q_x x) F_f^{(\varepsilon_1)^*} \hat{\vec{p}}_{i\varepsilon_2} F_i^{(\varepsilon_2)}\, \mathrm{d}x \tag{39}$$

where ν represents the transition between different states.

$$\hat{\vec{p}}_{i\varepsilon} = \hat{p}_x, p_{iy}, p_{n_i E_i} + \varepsilon\hbar k_s \qquad \varepsilon = 0, \pm 1 \tag{40}$$

$$F_{nE}^{(\varepsilon)} = \begin{cases} S_{nE}(x) & \varepsilon = 0 \\ b_{n-4}^{(\varepsilon)} S_{n-4,E}(x) + b_{n-2}^{(\varepsilon)} S_{n-2,E}(x) + b_n^{(\varepsilon)} S_{n,E}(x) & \\ \quad + b_{n+2}^{(\varepsilon)} S_{n+2,E}(x) + b_{n+4}^{(\varepsilon)} S_{n+4,E}(x) & \varepsilon = \pm 1. \end{cases} \tag{41}$$

From equation (38) we can write the probability of transition as the sum of five terms given by

$$W_{fi} = \sum_{\nu} W_{fi}^{(\nu)}, \qquad \nu = 0, \pm 1, \pm 2. \tag{42}$$

5. Angular and spectral distributions

5.1. Matrix elements

To find the matrix elements we define the vector of polarization $\vec{e}_1$ in the plane having the wavevector $\vec{q}$ and the z-axis and a vector $\vec{e}_2 \perp \vec{e}_1$ in the plane having the axes x and y as in the previous case [7].

$$\begin{aligned} \vec{e}_1 &= (\cos\theta\cos\varphi, \cos\theta\sin\varphi, -\sin\theta) \\ \vec{e}_2 &= (-\sin\varphi, \cos\varphi, 0). \end{aligned} \tag{43}$$

The summation in equation (36) can be transformed into an integral form given by

$$W_{fi}^{(\nu)} = \frac{e^2}{2\pi\hbar} \int (|\vec{\alpha}_{fi}^{(\nu)} \cdot \vec{e}_1|^2 + |\vec{\alpha}_{fi}^{(\nu)} \cdot \vec{e}_2|^2) |\vec{q}|^{-1} \delta(\omega_{fi} - \omega)\, \mathrm{d}\vec{q} \tag{44}$$

where

$$\vec{\alpha}_{fi}^{(\nu)} = \delta_{\sigma_{iz},\sigma_{fz}} \delta_{p_{iy},p_{fy}+\hbar q_y} \delta_{P_{n_i E_i + \nu\hbar k_s}, p_{n_f E_f + \hbar q_z}} \vec{D}_{fi}^{(\nu)}. \tag{45}$$

From the above equations we can notice that a hypersonic field can induce five radiative transitions for the channelling radiation. Corresponding transition probabilities are given by $W_{fi}^{(\nu)}$. The case $\nu = 0$ is for channelling in the absence of a hypersonic field. Since $\nu = \pm 2$ are

Table 1. Photon frequencies for $\nu = \pm 1$ at direct and inverse transitions.

	ν	
$n_i - n_f$	-1	$+1$
$+1$ direct transition	$\omega = \frac{\omega_E\left\{\xi\left[1+\left(\frac{3V_1}{4V_0}\right)\alpha^2(n_i+n_f+1)\right]+2\right\}}{\xi(1-\beta\cos\theta)} = \omega'_{\mathrm{dir}}$ for any ξ	$\omega = \frac{\omega_E\left\{\xi\left[1+\left(\frac{3V_1}{4V_0}\right)\alpha^2(n_i+n_f+1)\right]-2\right\}}{\xi(1-\beta\cos\theta)} = \omega''_{\mathrm{dir}}$ $\xi\left[1+\left(\frac{3V_1}{4V_0}\right)\alpha^2(n_i+n_f+1)\right] > 2$
-1 inverse transition	$\omega = \frac{\omega_E\left\{2-\xi\left[1+\left(\frac{3V_1}{4V_0}\right)\alpha^2(n_i+n_f+1)\right]\right\}}{\xi(1-\beta\cos\theta)} = \omega_{\mathrm{inv}}$ $\xi\left[1+\left(\frac{3V_1}{4V_0}\right)\alpha^2(n_i+n_f+1)\right] < 2$	$\omega = -\frac{\omega_E\left\{\xi\left[1+\left(\frac{3V_1}{4V_0}\right)\alpha^2(n_i+n_f+1)\right]+2\right\}}{\xi(1-\beta\cos\theta)} < 0$ not realized

forbidden (see equations (27) and (38)) we are left with only three radiative transitions induced by hypersound.

For $\nu = 0$ one obtains [3, 14]

$$\vec{D}^{(0)}_{fi} \approx -\mathrm{i}x_{fi}\left(\frac{\Omega_{fi}}{c}, 0, q_x\beta\right) \tag{46}$$

with

$$x_{fi} = \alpha\left(\sqrt{\frac{n_i+1}{2}}\delta_{n_f,n_i+1} + \sqrt{\frac{n_f+1}{2}}\delta_{n_i,n_f+1}\right). \tag{47}$$

It remains to calculate the matrix elements for $\nu = \pm 1$. It is found that only transitions with $n_i - n_f = \pm 1$ are allowed. Also direct transitions are possible for $\xi[1+(\frac{3V_1}{4V_0})\alpha^2(n_i+n_f+1)] > 2$ and $\nu = \pm 1$. Direct as well as inverse transitions are possible for $\xi[1+(\frac{3V_1}{4V_0})\alpha^2(n_i+n_f+1)] < 2$ and $\nu = -1$. Table 1 shows the photon frequencies for $\nu = \pm 1$.

5.2. Transition probabilities

The transition probabilities and intensities for the case $\nu = 0$ (i.e., without the hypersonic field) are given by [3, 14]

$$\frac{\mathrm{d}W^{(0)}_{fi}}{\mathrm{d}\Omega} \approx \frac{e^2\omega_E^3 x_{fi}^2}{2\pi\hbar c^3(1-\beta\cos\theta)^4}[(1-\beta\cos\theta)^2 - (1-\beta^2)\sin^2\theta\cos^2\varphi] \tag{48}$$

$$\frac{\mathrm{d}W^{(0)}_{fi}}{\mathrm{d}\omega} \approx \frac{e^2\omega_E^2 x_{fi}^2}{\hbar c^3}\left[1 - 2\frac{\omega}{\omega_m} + 2\left(\frac{\omega}{\omega_m}\right)^2\right] \tag{49}$$

$$\frac{\mathrm{d}I^{(0)}_{fi}}{\mathrm{d}\Omega} \approx \frac{e^2\omega_E^4 x_{fi}^2}{2\pi c^3(1-\beta\cos\theta)^5}[(1-\beta\cos\theta)^2 - (1-\beta^2)\sin^2\theta\cos^2\varphi] \tag{50}$$

$$\frac{\mathrm{d}I^{(0)}_{fi}}{\mathrm{d}\omega} \approx 3I^{(0)}_{fi}\frac{\omega}{\omega_m^2}\left[1 - 2\frac{\omega}{\omega_m} + 2\left(\frac{\omega}{\omega_m}\right)^2\right] \tag{51}$$

where

$$I^{(0)}_{fi} = \frac{4}{3}\frac{e^2\omega_E^4\gamma^4 x_{fi}^2}{c^3}. \tag{52}$$

We shall now analyse those transitions induced by hypersound, proceeding in the same way as in the previous case [7].

Substituting $q_x = (\omega \sin\theta \cos\varphi)/c$ and using equations (31) and (41),

$$\vec{D}_{fi}^{(\pm 1)} \approx \pm \frac{\mathrm{i} x_{fi}\mu\xi}{8c}\left[\frac{\sqrt{6}t}{4(2\xi \mp 1)} + \frac{[\omega_E + 6t]}{(\xi \mp 1)}\right]\left\{\xi\left[1 + \left(\frac{3V_1}{4V_0}\right)\alpha^2(n_i + n_f + 1)\right] \mp 2\right\} \times \left[1, 0, \frac{\beta \sin\theta \cos\varphi}{1 - \beta\cos\theta}\right] \tag{53}$$

for direct transitions, and

$$\vec{D}_{fi}^{(-1)} \approx \frac{\mathrm{i} x_{fi}\mu\xi}{8c}\left[\frac{\sqrt{6}t}{4(2\xi - 1)} + \frac{[\omega_E + 6t]}{(\xi - 1)}\right]\left\{2 - \xi\left[1 + \left(\frac{3V_1}{4V_0}\right)\alpha^2(n_i + n_f + 1)\right]\right\} \times \left[1, 0, \frac{\beta \sin\theta \cos\varphi}{1 - \beta\cos\theta}\right] \tag{54}$$

for inverse transitions.

Performing integration over ω in equation (44) and using equations (45), (53) and (54), we get

$$\frac{\mathrm{d}W_{fi}^{(\pm 1)}}{\mathrm{d}\Omega} \approx q_{3fi}^{(\pm 1)} \frac{e^2\omega_E^3 x_{fi}^2}{2\pi\hbar c^3(1 - \beta\cos\theta)^4}[(1 - \beta\cos\theta)^2 - (1 - \beta^2)\sin^2\theta\cos^2\varphi] \tag{55}$$

where the x_{fi} are given by

$$x_{n,n-1} = \alpha\left(\frac{n}{2}\right)^{1/2}\left(1 - n\frac{3}{4}\frac{V_1}{V_0}\alpha^2\right)$$

and

$$x_{n,n+1} = \alpha\left(\frac{n+1}{2}\right)^{1/2}\left(1 - (n+1)\frac{3}{4}\frac{V_1}{V_0}\alpha^2\right)$$

and the q_{sfi} are given by

$$q_{s_{fi}}^{(\pm 1)}(\xi) = \frac{\mu^2\xi^{(4-s)}\left\{\xi\left[1 + \left(\frac{3V_1}{4V_0}\right)\alpha^2(n_i + n_f + 1)\right] \mp 2\right\}^s}{64}\left[\frac{\sqrt{6}b}{4(2\xi \mp 1)} + \frac{[1 + 6b]}{(\xi \mp 1)}\right]^2 \tag{56}$$

for direct transitions, and

$$q_{s_{fi}}^{(-1)}(\xi) = \frac{\mu^2\xi^{(4-s)}\left\{2 - \xi\left[1 + \left(\frac{3V_1}{4V_0}\right)\alpha^2(n_i + n_f + 1)\right]\right\}^s}{64}\left[\frac{\sqrt{6}b}{4(2\xi - 1)} + \frac{[1 + 6b]}{(\xi - 1)}\right]^2 \tag{57}$$

for inverse transitions, with $b = \frac{t}{\omega_E}$.

Performing integration over the solid angle in equation (44), we get

$$\frac{\mathrm{d}W_{fi}^{(\pm 1)}}{\mathrm{d}\omega} \approx q_{2fi}^{\pm 1}\frac{e^2\omega_E^2 x_{fi}^2}{\hbar c^3}\left[1 - 2\frac{\omega}{\omega_{\pm fi}} + 2\left(\frac{\omega}{\omega_{\pm fi}}\right)^2\right] \tag{58}$$

where

$$\omega_{\pm fi} = 2\gamma^2\omega_E \begin{cases} \left[1 + \left(\frac{3V_1}{4V_0}\right)\alpha^2(n_i + n_f + 1)\right] - \frac{2}{\xi} & \nu = +1 \\ n_i - n_f = +1 \quad \xi\left[1 + \left(\frac{3V_1}{4V_0}\right)\alpha^2(n_i + n_f + 1)\right] > 2 & \\ \frac{2}{\xi} + \left[1 + \left(\frac{3V_1}{4V_0}\right)\alpha^2(n_i + n_f + 1)\right] & \nu = -1 \\ n_i - n_f = +1 & \\ \frac{2}{\xi} - \left[1 + \left(\frac{3V_1}{4V_0}\right)\alpha^2(n_i + n_f + 1)\right] & \nu = -1 \\ n_i - n_f = -1 \quad \xi\left[1 + \left(\frac{3V_1}{4V_0}\right)\alpha^2(n_i + n_f + 1)\right] < 2. & \end{cases} \tag{59}$$

Multiplying equations (55) and (58) by $\hbar\omega$, we get the angular and spectral distribution of the radiation intensity as

$$\frac{\mathrm{d}I_{fi}^{(\pm 1)}}{\mathrm{d}\Omega} \approx q_{4fi}^{(\pm 1)} \frac{e^2\omega_E^4 x_{fi}^2}{2\pi(1-\beta\cos\theta)^5}[(1-\beta\cos\theta)^2 - (1-\beta^2)\sin^2\theta\cos^2\varphi] \tag{60}$$

$$\frac{\mathrm{d}I_{fi}^{(\pm 1)}}{\mathrm{d}\omega} \approx 3I_{fi}^{(\pm 1)} \frac{\omega}{\omega_{\pm fi}^2}\left[1 - 2\frac{\omega}{\omega_{\pm fi}} + 2\left(\frac{\omega}{\omega_{\pm fi}}\right)^2\right] \tag{61}$$

where

$$I_{fi}^{(\pm 1)} = \frac{4}{3}\frac{e^2\omega_E^4\gamma^4 x_{fi}^2}{c^3} q_{4fi}^{(\pm 1)}. \tag{62}$$

The expression for total intensity is proportional to the power of hypersound ($\sim\mu^2$). $q_{4fi}^{(\pm 1)} \ll 1$ for $n_i - n_f = +1$. Comparing with the results in the harmonic case [7], the angular and spectral distributions of radiations differ by the anharmonic parameters given in equation (56).

6. Inverse radiative transitions

Inverse radiative transitions $n_i - n_f = -1$ are excited by the hypersound. From equation (57) it is obvious that as $\xi \to 1$, $q_{4fi}^{(-1)} \gg 1$. This shows that there is a resonant amplification of the channelling radiation intensity due to inverse transitions which is also an amplification of the radiation in the harmonic case. Figure 1 shows the influence of both hypersound and anharmonicity. The curves for a specific case $n_i = 0, n_f = 1$ are calculated using equation (61) for inverse transitions with $\nu = -1, \mu = 0.1$ and equation (51) for direct transitions with $\nu = 0((s_{fi})^{-1} = 2e^2\omega_E^3\gamma^2 x_{fi}^2/c^3)$. For the sake of comparison with the harmonic case [7], we take $\xi = 1.01$ and $\xi = 1.03$. The ratio of spectral distribution of radiation intensity in the anharmonic case to that in the harmonic case is about 1.16, which shows that the effect of the anharmonic term cannot be neglected.

7. Conclusions

In the present work, we have studied the effects of a hypersonic field on the positron planar channelling radiation. The anharmonic effects (quartic term) of the interplanar transverse potential seen by the positron have been included in the problem. The corresponding eigenspectrum is calculated from the Dirac equation. The wavefunction of the positron gets

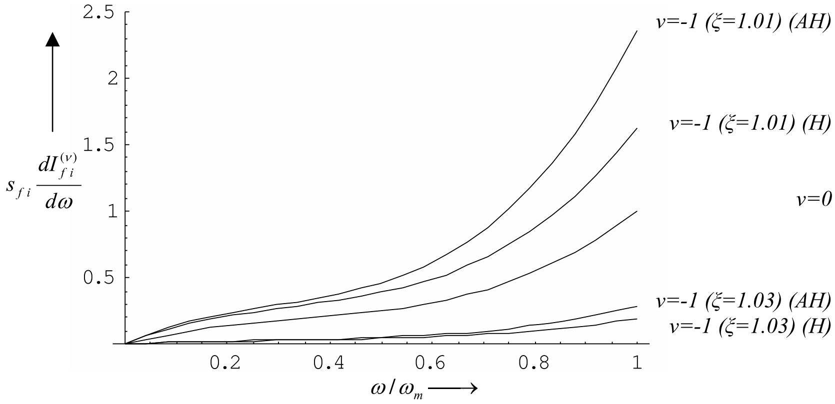

Figure 1. Spectral distributions in the case of inverse radiative transitions at $\nu = -1$ and $\xi = 1.01$ and 1.03 for harmonic and anharmonic cases and at $\nu = 0$.

modified by these effects and changes the observable parameters like frequency and intensity of radiation. The fractional change in the frequency of the emitted radiation from that in the harmonic case is found to be directly proportional to the strength of the anharmonic term. We find considerable variation of radiation intensity due to the anharmonic effects as seen from figure 1: an increase by a factor 1.16 over harmonic case. This intensity amplification shows that the anharmonic terms cannot be neglected.

We also find that the amplitudes (equation (31)), responsible for the intensity of the emitted radiation, show a resonance when the hypersonic field wavelength λ_s approaches a value $\pi/2\varsigma_{\mathrm{ah}}\omega_E$ which is exactly half of the corresponding quantity for the harmonic case. An experimental verification of these findings on the effects of hypersonic fields on channelling radiation is required to give impetus to further research in this exciting field.

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